Physics of thrust prediction of the solar wind electric sail propulsion system

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Abstract. The electric solar wind sail is a newly invented way for using the solar wind dynamic pressure for providing thrust for a spacecraft. An electric sail spacecraft deploys long, thin, conducting tethers which are centrifugally stretched and kept in a high positive potential by an onboard electron gun. Scientifically, the essential problem is to predict the thrust force per unit length that the plasma stream exerts on the tether when the tether is kept in a 15-40 kV voltage. Recently, theoretical arguments were put forward which suggest that trapped electrons are almost completely absent from the electric sail. In view of these arguments, we make here the assumption that trapped electrons are absent and proceed to derive a simplified analytical thrust formula. This formula predicts about five times higher electric sail thrust than the original estimates which included trapped electrons.

1. Introduction

The electric solar wind sail spacecraft propulsion system (Janhunen 2004; Janhunen and Sandroos 2007; Janhunen 2009a,b) consists of long, thin, conducting, charged and centrifugally stretched tethers (Fig. 1). The positive charging of the tethers is maintained by a continuously operating onboard electron gun. The electric sail presents itself as a technically feasible way of using the natural solar wind dynamic pressure for producing spacecraft thrust and thereby achieving the capability to move rather freely in the solar system without consuming propellant.

The underlying elementary process of the solar wind electric sail propulsion system is the interaction of the solar wind plasma stream with a positively charged tether. The computational problem is to predict the thrust force per unit length that the plasma stream exerts on the tether when the tether is artificially kept in a 15-40 kV voltage. Since the force per unit length is approximately equal to the sheath width multiplied by the solar wind dynamic pressure, the problem is closely related to the prediction of the collisionless electron sheath width in flowing plasma conditions, a problem tackled recently e.g. by Sanmartín et al. (2008).

The first attempts to predict the force used time-accurate particle-in-cell (PIC) simulations (Janhunen and Sandroos 2007). By construction, these simulations included the extra shielding effect of trapped electron population which is always formed when the potential is ramped up. This shielding reduces the sheath width and the tether's thrust.

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Figure 1. Schematic presentation of the electric solar wind sail deep-space propulsion method.

The purpose of this paper is to analyse the electric sail thrust prediction in a better way and to derive an approximate thrust formula using recently discovered theoretical arguments.

2. Absence of trapped electrons

When the tether's potential is first ramped up, some initially positive energy electrons get trapped by the deepening potential well (Janhunen and Sandroos 2007). Perhaps somewhat surprisingly, the number of trapped electrons does not depend on the rate at which the potential is ramped up (Janhunen 2009b). In case of a purely 2-D infinitely long tether, the trapped electron population which is orbiting the tether appears to be very long-lived, since collisional processes and radiative losses have only a very minor importance. Thus there appears to be no process that could deplete the trapped electron population in any reasonable timescale.

Consider an electron spiralling around a long tether (Fig. 2). The electron's path in the perpendicular plane is qualitatively somewhat similar to planetary orbits except that the perihelion angle of the orbit changes rapidly. As long as the tether's potential is two-dimensional, the angular momentum and parallel velocity along the tether are conserved quantities, as is the total energy. These conserved quantities reflect the non-dependence of the potential on the parallel coordinate, the azimuthal coordinate and time, respectively.



Figure 2. Typical orbit of an electron in the perpendicular plane around the attractive tether after shorter (a) and longer (b) integration.



Figure 3. Starfish-shaped potential structure of multi-tether electric sail and typical electron orbit around a tether

The particle always has some finite velocity along the tether (Fig. 3). If it originally propagates toward the tip of the tether (i.e. away from the spacecraft), it gets reflected at the tip and then propagates toward the spacecraft. Therefore each trapped electron periodically visits the vicinity of the spacecraft. Near the spacecraft the potential is no longer symmetric, but instead has a complicated 3-D shape, resembling a starfish (Fig. 3). Near the spacecraft, therefore, the angular momentum and parallel velocity of the particle are effectively randomised, and the particle generally starts to move along a different tether away from the spacecraft after the interaction. The randomisation implies that there is a small but finite probability that the electron's new angular momentum will be small enough that it flies through the tether wires each time it orbits the tether. For micrometeoroid tolerance, the tether is not made from a single wire, but from typically four wires forming a so-called Hoytether structure, Fig. 4, (Hoyt and Forward 2001). The end result is that the electron has a small likelihood of hitting one of the tether wires and thereby be lost.

The speed of electrons is so high that although the probability per orbit that the electron will collide the wire is small, the lifetime of a trapped electron against this mechanism is typically only a few minutes (Janhunen 2009b). In other words, the potential pattern around the multi-tethered electric sail spacecraft is not symmetrical enough to allow for long-time trapping of electrons. The electrons will leak out from the trap by finding a way to collide with the tether Janhunen



Figure 4. A four-wire Hoytether. Wire bonding sites are shown by dots.

in a timescale which is rapid compared to the operational time of the propulsion system.

This consideration strongly suggests that trapped electrons are effectively absent from the electric sail except during a few minute transient period after the potential was switched on or increased.

3. Thrust prediction and electrosphere

Based on the assertion that trapped electrons are absent, implicit equations from which the thrust can be numerically solved were derived in Janhunen (2009b). Here we shall present a simplified, more approximate derivation which produces a convenient closed-form result. Motivated by the above considerations, we shall assume that trapped electrons are absent.

Consider a positively charged tether with voltage V_0 , embedded in perpendicularly flowing solar wind having number density n_0 and velocity v. Solar wind ions are repelled by the tether so ion density is zero up to some radial distance R from which is gradually increases to the ambient value. Solar wind electrons are attracted by the tether, but due to the conservation of angular momentum their average density is much smaller than ambient when r < R, see e.g. (Sanmartín et al. 2008) and (Janhunen 2009b). Therefore it is not a very bad approximation to assume that the electron density is zero when r < R. In other words, in this approximation both ion and electron density are zero up to r = R and they have the ambient value n_0 for r > R. Then the electric field around the tether is simply obtained the vacuum expression

$$E(r) = \frac{\lambda}{2\pi r\epsilon_{\rm o}} \tag{1}$$

where λ is the line charge of the tether (ampere per metre). The distance R is determined from the fact that at R, the pressure of the electric field $(1/2)\epsilon_0 E^2$ must be equal to the solar wind dynamic pressure $P_{\rm dyn} = n_0 m_p v^2$ where m_p is the proton mass. From these we obtain

$$R = \frac{\lambda}{2\pi\sqrt{2\epsilon_{\rm o}P_{\rm dyn}}}.$$
(2)

The thrust per unit length dF/dz is then given by $dF/dz = KRP_{dyn}$ where $K \approx 3.09$ is a normalisation factor determined from test particle calculation (Janhunen and Sandroos 2007).

The only remaining task is to relate the line charge λ to the tether potential V_0 . Applying formula A4 of Janhunen and Sandroos (2007) we have

$$\lambda = \frac{2\pi\epsilon_{\rm o}(V_0 + V_1)}{\ln(R/r_w^*)} \tag{3}$$

where r_w^* is the effective electric width of the tether (Janhunen and Sandroos 2007), typically $r_w^* \sim 1$ mm. A rigorous lower bound for a multiline tether r_w^* is the two-wire tether result $r_w^* = \sqrt{r_w h}$ where r_w is the physical wire radius and h the tether width. We have added the solar wind proton kinetic energy term $V_1 = (1/2)m_p v^2/e$ to the potential because in order to deflect the protons, the potential must be equal to V_1 at r = R. After substituting everything we obtain the thrust per unit length

$$\frac{dF}{dz} \approx \frac{2.2}{\ln(R/r_w^*)} (V_0 + V_1) \sqrt{\epsilon_0 P_{\rm dyn}} = \frac{2.2}{\ln(R/r_w^*)} (V_0 + V_1) v \sqrt{\epsilon_0 n_0 m_p}.$$
 (4)

Typically $eV_0 \ll (1/2)m_p v^2 \approx 1$ keV so that the V_1 can be usually dropped. The quantity R is in principle unknown and should be found by iterating the equations, but since the dependence on R is only logarithmic, a reasonably good approximation can be obtained by setting e.g. $R \approx 100$ m.

In analog with the magnetosphere, the term *electrosphere* was introduced by Janhunen (2009b) to describe the situation. The force balance equation at r = R is analogous to the magnetic force balance equation at the subsolar point of the terrestrial and other magnetospheres which determines the subsolar distance of the magnetopause. Analogously, it is natural to call the electrosphere boundary at r = R the *electropause*.

Fig. 5 shows a comparison of the approximate formula (4) with the more accurate (and still approximate) formula of Janhunen (2009b). We see that under the assumption of zero electron density the linearised formula (4) is in quite good agreement with the more accurate result. Even if the average electron density inside the electrosphere would be equal to the ambient density or larger, the result still remains qualitatively valid.

4. Conclusions

We have reviewed recent results which suggest that trapped electrons are completely or nearly absent from the electric sail. The resulting thrust per length values are ~ 5 times higher than those reported earlier based on PIC simulations (Janhunen and Sandroos 2007). Under the assumption of no trapped electrons, we derived a simple approximate formula for predicting the thrust per length and showed that it is in good agreement with more complicated formulas presented earlier. With the improved thrust estimates, the promise of practical usefulness of the electric sail concept for solar system transportation appears to be high.

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Figure 5. The approximate thrust per length, Eq. (4) (dashed) with more accurate results from Janhunen (2009b), with different values for average electron density inside the electrosphere. R = 100 m was assumed.

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